27.

$$\int x(1+x)^{10}\,dx = x\cdot\frac{1}{11}(1+x)^{11} - \int \frac{1}{11}(1+x)^{11}\,dx = \frac{1}{11}x(1+x)^{11} - \frac{1}{11}\int (1+x)^{11}\,dx =$$

$$u = x \Rightarrow du = dx$$

$$dv = (1+x)^{10}\,dx \Rightarrow v = \int (1+x)^{10}\,dx = \int z^{10}\,dz = \frac{z^{11}}{11} = \frac{1}{11}(1+x)^{11}$$

$$z = (1+x) \Rightarrow dz = dx$$

$$\frac{1}{11}x(1+x)^{11} - \frac{1}{11}\cdot\frac{(1+x)^{12}}{12} = \frac{1}{11}(1+x)^{11}\left(x - \frac{1}{12}(1+x)\right)$$

$$u = (1+x) \Rightarrow du = dx \Rightarrow \int u^{11}\,du = \frac{u^{12}}{12} = \frac{(1+x)^{12}}{12}$$

28.

$$\int \frac{3x}{e^x}\,dx = \int 3xe^{-x}\,dx = 3x\cdot -e^{-x} - \int -e^{-x}\cdot 3dx = -3e^{-x} + 3\int e^{-x}\,dx = -3xe^{-x} + 3\cdot -e^{-x} = -3xe^{-x} - 3e^{-x} = -3e^{-x}(x+1)$$

$u = 3x \Rightarrow du = 3dx$

$dv = e^{-x}dx \Rightarrow v = \int e^{-x}dx = -\int e^{z}dz = -e^{z} = -e^{-x}$

$z = -x \Rightarrow dz = -dx \Rightarrow -dz = dx$

Use the same process

29.

$$\int x(x+5)^{-3}\,dx = x\cdot\frac{(x+5)^{-2}}{-2} - \int \frac{(x+5)^{-2}}{-2}\,dx = -\frac{1}{2}x(x+5)^{-2} + \frac{1}{2}\int (x+5)^{-2}\,dx =$$

$$u = x \Rightarrow du = dx$$

$$dv = (x+5)^{-3}\,dx \Rightarrow v = \int (x+5)^{-3}\,dx = \int z^{-3}\,dz = \frac{z^{-2}}{-2} = \frac{(x+5)^{-2}}{-2}$$

$$z = (x+5) \Rightarrow dz = dx$$

$$-\frac{1}{2}x(x+5)^{-2} + \frac{1}{2}\cdot -(x+5)^{-1} = -\frac{1}{2}(x+5)^{-2}\left(x + (x+5)\right) = -\frac{1}{2}(x+5)^{-2}\left(2x+5\right)$$

$$u = (x+5) \Rightarrow du = dx \Rightarrow \int u^{-2}\,du = \frac{u^{-1}}{-1} = -(x+5)^{-1}$$

30.

$$\int \sqrt{x}\ln\sqrt{x}\,dx = \int x^{\frac{1}{2}}\ln x^{\frac{1}{2}}dx = \ln x^{\frac{1}{2}}\cdot\frac{2}{3}x^{\frac{3}{2}} - \int\frac{2}{3}x^{\frac{3}{2}}\cdot\frac{1}{2}x^{-1}dx = \frac{2}{3}x^{\frac{3}{2}}\ln x^{\frac{1}{2}} - \frac{1}{3}\int x^{\frac{1}{2}}dx = \frac{2}{3}x^{\frac{3}{2}}\ln x^{\frac{1}{2}} - \frac{1}{3}\cdot\frac{2}{3}x^{\frac{3}{2}}$$

$$u = \ln x^{\frac{1}{2}} \Rightarrow du = \frac{1}{x^{\frac{1}{2}}}\cdot\frac{1}{2}x^{-\frac{1}{2}}dx \Rightarrow du = \frac{1}{2}x^{-1}dx$$

Same process used

$$dv = x^{\frac{1}{2}}dx \Rightarrow v = \int x^{\frac{1}{2}}dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3}x^{\frac{3}{2}}$$

$$\frac{2}{3}x^{\frac{3}{2}}\ln x^{\frac{1}{2}} - \frac{2}{9}x^{\frac{3}{2}} = \frac{2}{3}x^{\frac{3}{2}}\left(\ln x^{\frac{1}{2}} - \frac{1}{3}\right)$$

31.

$$\int x^{-3}\ln x\,dx = -\frac{1}{2}x^{-2}\ln x - \int -\frac{1}{2}x^{-2}\cdot\frac{1}{x}dx = -\frac{1}{2}x^{-2}\ln x + \frac{1}{2}\int x^{-3}dx = -\frac{1}{2}x^{-2}\ln x + \frac{1}{2}\cdot -\frac{1}{2}x^{-2} =$$

$$u = \ln x \Rightarrow du = \frac{1}{x}dx$$

Same process used

$$dv = x^{-3}dx \Rightarrow v = \int x^{-3}dx = \frac{x^{-2}}{-2} = -\frac{1}{2}x^{-2}$$

$$-\frac{1}{2}x^{-2}\ln x - \frac{1}{4}x^{-2} = -\frac{1}{2}x^{-2}\left(\ln x + \frac{1}{2}\right)$$

32.

$$\int x^5\sin(x^3)dx = \int x^3\cdot x^2\sin(x^3)dx = x^3\cdot -\frac{1}{3}\cos\left(x^3\right) - \int -\frac{1}{3}\cos\left(x^3\right)\cdot 3x^2dx = -\frac{1}{3}x^3\cos(x^3) + \int x^2\cos(x^3)dx =$$

$$u = x^3 \Rightarrow du = 3x^2dx$$

$$dv = x^2\sin(x^3)dx \Rightarrow v = \int x^2\sin(x^3)dx = \frac{1}{3}\int\sin z\,dz = -\frac{1}{3}\cos z = -\frac{1}{3}\cos\left(x^3\right)$$

$$z = x^3 \Rightarrow dz = 3x^2dx \Rightarrow \frac{1}{3}dz = x^2dx$$

$$-\frac{1}{3}x^3\cos(x^3) + \frac{1}{3}\sin\left(x^3\right) = \frac{1}{3}\left(x^3\cos(x^3) + \sin(x^3)\right)$$

$$u = x^3 \Rightarrow du = 3x^2dx \Rightarrow \frac{1}{3}du = x^2dx \Rightarrow \frac{1}{3}\int\cos u\,du = \frac{1}{3}\sin u = \frac{1}{3}\sin\left(x^3\right)$$